

Calculus Warm Up #12-2

1) find $\frac{dy}{dx}$

$\sin(x + y) = x$

2) solve for x

$\sqrt{2x} = \sin(\arccos\sqrt{x})$

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1) find $\frac{dy}{dx}$

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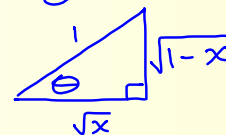
$$\cos(x+y)\left(1 + \frac{dy}{dx}\right) = 1$$

$$1 + \frac{dy}{dx} = \frac{1}{\cos(x+y)}$$

$$\boxed{\frac{dy}{dx} = \sec(x+y) - 1}$$

2) solve for x

$\sqrt{2x} = \sin(\underbrace{\arccos\sqrt{x}}_{\theta})$



$$\sqrt{2x} = \sin \theta$$

$$\sqrt{2x} = \sqrt{1-x} \quad ; \quad 0 \leq x \leq 1$$

$$2x = 1 - x$$

$$3x = 1$$

$$\boxed{x = \frac{1}{3}}$$

HW Questions: p. 693

In Exercises 1–10, find dy/dx and d^2y/dx^2 , and evaluate each at the specified value of the parameter.

<u>Parametric equations</u>	<u>Point</u>
1. $x = 2t, y = 3t - 1$	$t = 3$
3. $x = t + 1, y = t^2 + 3t$	$t = -1$
5. $x = 2 \cos \theta, y = 2 \sin \theta$	$\theta = \frac{\pi}{4}$

7. $x = 2 + \sec \theta, y = 1 + 2 \tan \theta \quad \theta = \frac{\pi}{6}$

$$\frac{dy}{dx} = \frac{2 \sec^2 \theta}{\sec \theta \tan \theta}$$

$$\frac{dy}{dx} = \frac{2 \sec \theta}{\tan \theta}$$

$$\frac{dy}{dx} = \frac{2}{\frac{\cos \theta}{\sin \theta}}$$

$$\boxed{\frac{dy}{dx} = 2 \csc \theta}$$

When $\theta = \frac{\pi}{6}$

$$\frac{dy}{dx} = 2 \csc \frac{\pi}{6} = 2\left(\frac{2}{1}\right) = \boxed{4}$$

$$\frac{dy}{dx} = 2 \csc \theta$$

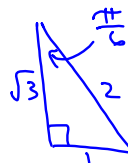
$$\frac{d^2y}{dx^2} = \frac{-2 \csc \theta \cot \theta}{\sec \theta \tan \theta}$$

$$\frac{dx}{d\theta}$$

$$= -2 \cot^3 \theta$$

When $\theta = \frac{\pi}{6}$

$$\frac{d^2y}{dx^2} = -2(\sqrt{3})^3 = -6\sqrt{3}$$



9. $x = \cos^3 \theta$, $y = \sin^3 \theta$

$\theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{3(\sin \theta)^2 (\cancel{\cos \theta})}{3(\cancel{\cos \theta})^2 (-\cancel{\sin \theta})} \quad y = (\sin \theta)^3$$

$$\frac{dy}{dx} = - \frac{\sin \theta}{\cos \theta}$$

$$\frac{dy}{dx} = - \tan \theta$$

when $\theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = - \tan \frac{\pi}{4}$$

$$= \boxed{-1}$$

In Exercises 11–16, find an equation of the tangent line to the curve at the specified value of the parameter.

Parametric equations

Point

11. $x = 2t$, $y = t^2 - 1$

$t = 2$

13. $x = t^2 - t + 2$, $y = t^3 - 3t$

$t = -1$

15. $x = 2 \cot \theta$, $y = 2 \sin^2 \theta$

$\theta = \frac{\pi}{4}$

In Exercises 19–28, find all points (if any) of horizontal and vertical tangency.

19. $x = 1 - t$

$y = t^2$

$$\frac{dy}{dx} = \frac{2t}{-1} \rightarrow 2t = 0$$

$$t = 0$$

horiz
tangent
 $(1, 0)$

21. $x = 1 - t$

$y = t^3 - 3t$

$$x = 1$$

$$y = 0$$

No vertical tangent

23. $x = 3 \cos \theta$

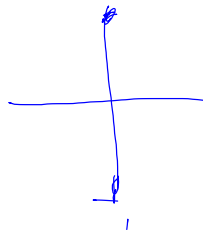
$y = 3 \sin \theta$

$$\frac{dy}{dx} = \frac{3 \cos \theta}{-3 \sin \theta} \rightarrow \text{horiz tangent}$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2} + \pi n$$

$$(0, 3) \quad (0, -3)$$



25. $x = 4 + 2 \cos \theta$

$y = -1 + \sin \theta$

27. $x = \sec \theta$

$y = \tan \theta$

HW: finish AP WS # 8

(Pink)

8 can be turned in tomorrow for +1 EC,
or due Monday after break.

Answers posted tonight.

AP Rev. #8

1a) f' is decreasing on $1.7 < x < 1.9$
so f'' is negative. f will be
concave down.

b) Check endpoints & where $f' = 0$
to justify answer. $x = \sqrt{\pi}$

2a) $f'(22) = -3 \text{ Cal/min}^2$

b) looking for max of f'

use $f''(t) = -\frac{3}{2}t + 3$

\rightarrow f''
 $\leftarrow \oplus \quad 2 \quad \ominus \rightarrow$
 confirms max of f'
 @ $t=2$

$$3b) \quad c = 1$$

$$5a) \quad g'(0) = a - 4$$

$$g''(0) = a^2 + 3$$

$$b) \quad h'(x) = f'(x) \cos kx - k \sin kx$$

$$\text{slope} = h'(0) = -4$$

pt of tangency $(0, 2)$

$$\boxed{y = -4x + 2}$$